FoDA:

L2

Probability Review #1

Probability

Sample space Ω \in

sample outcome $\omega \in \Omega$

events $A \subseteq \Omega$

Example 6-sided die

$\Omega = \{1, 2, 3, 4, 5, 6\}$

$\omega = 3$

event "even" $A = \{2, 4, 6\} \subset \Omega$

"odd" $B = \{1, 3, 5\} \subset \Omega$

$$\Pr(A) = \frac{|\{2,4,6\}|}{|\Omega|} = \frac{3}{6} = \frac{1}{2}$$

Probability $P(A)$

- $0 \le Pr(A) \le 1$
- $Pr(\Omega) = 1$
- disjoint sets $A_1, A_2, \ldots$

$$i \neq j \quad A_i \cap A_j = \phi$$

$$Pr\left(\bigcup_{i=1} A_i\right) = \sum_{i=1} Pr(A_i)$$

Biased Coin

$$\Omega = \{H, T\}$$

$$A_1 = \{H\} \qquad A_2 = \{T\}$$

$$Pr(A_1) = Pr(H) = 0.6$$
$$Pr(A_2) = Pr(T) = 0.4$$

$$Pr(\Omega) = 1$$

$$Pr(A_1) + Pr(A_2) = 0.6 + 0.4 = 1$$

Continuous Sample Spaces

- water, land, time $\mathbb{R}$

Train leave Zurich

1:37

$\Omega = [1{:}37{:}00,\ 1{:}38{:}00)$

event A first 40 seconds

$= [1{:}37{:}00,\ 1{:}37{:}40)$

$\Pr(A) = 0.8$

Random Variable

X variable, not yet set
outcome will be determined
by some random process

$X: \Lambda \to \Omega$

measurable function

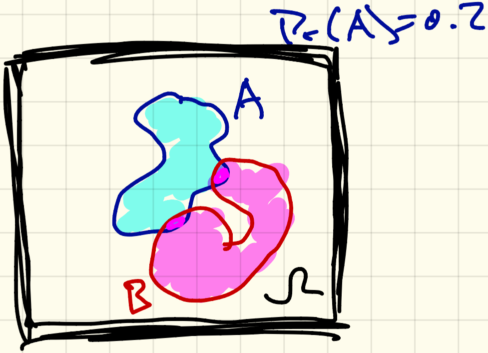
sample spaces

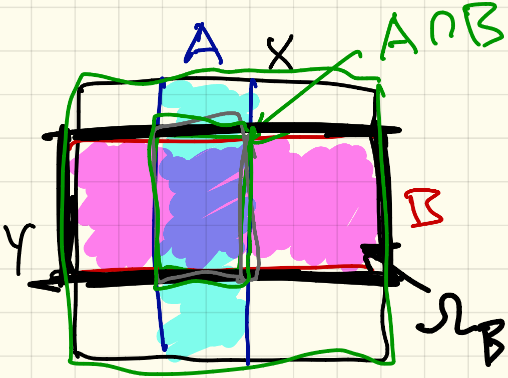
common $\Omega \subset \mathbb{R}$

$\Lambda = \{H, T\}$

$X(H) = 1$
$X(T) = 4$

$\Omega = \{1, 4\}$





Conditional Prob

$$\Pr(A \mid B)$$

given

Prob A, assuming event B = true

$$\Pr(A \mid B) = \frac{\Pr(A \cap B)}{\Pr(B)}$$

Two **events** A, B <u>independent</u>

$$\Pr(A \mid B) = \Pr(A)$$

or

$$\Pr(B \mid A) = \Pr(B)$$

or

$$\Pr(A \cap B) = \Pr(A) \cdot P(B)$$

Two random variables (RVs) X, Y.

<u>independent</u> iff all $A \subset \Omega_X$
$B \subset \Omega_Y$

$$\Pr(A \cap B) = \Pr(A) \cdot \Pr(B)$$

Example: Two Random Variables

$$T = \begin{cases} 1 & \text{if test is positive} \\ 0 & \text{if test is negative} \end{cases}$$

$$C = \begin{cases} 1 & \text{if have covid-19} \\ 0 & \text{if don't have covid-19} \end{cases}$$

	$C=1$	$C=0$	$\Pr(T=1)$
$T=1$	0.01	0.001	$\to$ 0.0101
$T=0$	0.0001	0.9889	

$$\Pr(C=1 \mid T=1) = \frac{\Pr(C=1 \wedge T=1)}{\Pr(T=1)} = \frac{0.01}{0.0101} = \frac{100}{101} \approx 99\%$$

$$\overset{?}{=} \Pr(C=1) \neq 0.0101$$

not independent

Density functions

continuous Random variables X

outcome $k \in \Omega$

event $A \subset \Omega$

$$\Pr(X = k) = 0$$

ok

$$\Pr(X \in A) \neq 0 \quad \text{or} = 0$$

probability $\Longrightarrow$ probability density function

pdf

likelihood

$$\Pr(X=\omega) \neq f_X(\omega) \qquad f_X : \Omega \to \mathbb{R}_{\geq 0}$$

$$\Pr(X \in A) = \int_{\omega \in A} f_X(\omega)\, d\omega$$

Cummolative density function

cdf

$\Omega = \mathbb{R}$

$$F_X(t) = \int_{w=-\infty}^{t} f_X(w)\, \partial w$$

$$f_X(w) = \frac{\partial F_X(w)}{\partial w}$$